Special Practice Problems

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~[JEE (Mains & Advanced)]~

Differential Calculus & Integral Calculus

"We should not give up and we should not allow the problem to defeat us."-A P J ABDUL KALAM

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. Tangents are drawn from the origin to the curve $y = \sin x$, then their point of contact lie on the curve

(a) $x^2 + y^2 = 1$ (b) $x^2 - y^2 = 1$

(c) $\frac{1}{x^2} + \frac{1}{y^2} = 1$ (d) $\frac{1}{y^2} - \frac{1}{x^2} = 1$

2. The locus of all points on the curve $y^2 = 4a\left(x + a\sin\left(\frac{x}{a}\right)\right)$ at which the tangent is parallel to x-axis is

(a) a straight line (b) a circle

(c) a parabola (d) an ellipse

3. The angle of intersection of curves $y = [|\sin x| + |\cos x|]$ and $x^2 + y^2 = 5$, where [.] denotes the greatest integer function, is

(a) $\tan^{-1} 2$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}(\sqrt{2})$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

4. If the normal to the curve $y = f(x)$ at $x = 0$ be given by the equation $3x - y + 3 = 0$, then the value of $\lim_{x \to 0} x^2 \{f(x^2) - 5f(4x^2) + 4f(7x^2)\}^{-1}$ is

(a) $-1/5$ (b) $-1/4$

(c) $-1/3$ (d) $-1/2$

5. The slope of the normal at the point with abscissa $x = -2$ of the graph of the function $f(x) = |x^2 - |x||$ is

(a) $-1/6$ (b) $-1/3$

(c) $1/6$ (d) $1/3$

6. A spherical balloon is pumped at the rate of 10 inch³/ min, the rate of increase of its radius if its radius is 15 inch is

(a) $\frac{1}{30\pi}$ inch/min (b) $\frac{1}{60\pi}$ inch/min

(c) $\frac{1}{90\pi}$ inch/min (d) $\frac{1}{120\pi}$ inch/min

7. A ladder 20 ft long has one end on the ground and the other end in contact with a vertical wall. The lower end slips along the ground. If the lower end of the ladder is 16 ft away from the wall, upper end is moving λ times as fast as the lower end, then λ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

8. A kite is moving horizontally at a height of 151.5 m. If the speed of the kite is 10 m/s, how fast is the string being let out, when the kite is 250 m from the boy who is flying the kite, the height of the boy being 1.5 m ?

(a) 4 m/s (b) 8 m/s

(c) 16 m/s (d) 32 m/s

9. The approximate value of square root of 25.2 is

(a) 5.01 (b) 5.02

(c) 5.03 (d) 5.04

10. The approximate value of $(0.007)^{1/3}$ is

(a) $\frac{21}{120}$ (b) $\frac{23}{120}$

(c) $\frac{29}{120}$ (d) $\frac{31}{120}$

11. If $y = \ln\left(\frac{x}{a+bx}\right)^x$, then $x^3 \frac{d^2y}{dx^2}$ is equal to

(a) $\left(\frac{dy}{dx}+x\right)^2$ (b) $\left(\frac{dy}{dx}-y\right)^2$

(c) $\left(x\frac{dy}{dx}+y\right)^2$ (d) $\left(x\frac{dy}{dx}-y\right)^2$

12. If $y = \left(\frac{ax+b}{cx+d}\right)$, then $2\frac{dy}{dx} \cdot \frac{d^3y}{dx^3}$ is equal to

(a) $\left(\frac{d^2y}{dx^2}\right)^2$ (b) $3\frac{d^2y}{dx^2}$

(c) $3\left(\frac{d^2y}{dx^2}\right)^2$ (d) $3\frac{d^2x}{dy^2}$

13. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to

(a) $(1+\ln x)^{-1}$ (b) $(1+\ln x)^{-2}$

(c) $\ln x\,(1+\ln x)^{-2}$ (d) none of these

14. If $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = k$, then k is equal to

(a) 0 (b) 1

(c) 2 (d) none of these

15. If $y = (1+x)(1+x^2)(1+x^4)\ldots.(1+x^{2^n})$, then $\frac{dy}{dx}$ at $x = 0$ is

(a) 0 (b) −1

(c) 1 (d) none of these

16. If $x = e^{y+e^{y+e^{y+e^{y+\ldots\infty}}}}$, then $\frac{dy}{dx}$ is

(a) $\frac{1}{x}$ (b) $\frac{1-x}{x}$

(c) $\frac{x}{1+x}$ (d) none of these

17. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is

(a) 0 (b) 1

(c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1+x^2}$

18. If $x^y \cdot y^x = 16$, then $\frac{dy}{dx}$ at (2, 2) is

(a) −1 (b) 0

(c) 1 (d) none of these

19. If $y^2 = P(x)$ is a polynomial of degree 3, then $2\frac{d}{dx}\left[y^3\frac{d^2y}{dx^2}\right]$ equals

(a) $P'''(x) + P'x$ (b) $P''(x) \cdot P'''(x)$

(c) $P(x) \cdot P'''(x)$ (d) none of these

20. If $5f(x) + 3f\left(\frac{1}{x}\right) = x + 2$ and $y = x\,f(x)$, then $\left(\frac{dy}{dx}\right)_{x=1}$ is equal to

(a) 14 (b) 7/8

(c) 1 (d) none of these

21. If $2^x + 2^y = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$ is

(a) 0 (b) −1

(c) 1 (d) 2

22. If $f(x) = |x-2|$ and $g(x) = fof(x)$, then for $x > 20$, $g'(x)$ is equal to

(a) 2 (b) 1

(c) 3 (d) none of these

23. If $f(x) = \sin^{-1}(\sin x) + \cos^{-1}(\sin x)$ and $\phi(x) = f(f(f(x)))$, then $\phi'(x)$ is equal to

(a) 1 (b) $\sin x$

(c) 0 (d) none of these

24. If $f(x) = (\log_{\cot x} \tan x)(\log_{\tan x} \cot x)^{-1} + \tan^{-1}\left(\frac{x}{\sqrt{(4-x^2)}}\right)$, then $f'(0)$ is equal to

(a) −2 (b) 2

(c) $\frac{1}{2}$ (d) 0

25. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $x^3y\frac{dy}{dx}$ equals

(a) − 1 (b) 0

(c) 1 (d) none of these

26. If variables x and y are related by the equation $x = \int_0^y \frac{1}{\sqrt{(1+9u^2)}}\,du$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{\sqrt{(1+9y^2)}}$ (b) $\sqrt{(1+9y^2)}$

(c) $(1+9y^2)$ (d) $\frac{1}{(1+9y^2)}$

27. If $y^{1/n} = \{x + \sqrt{(1+x^2)}\}$, then $(1+x^2)\,y_2 + xy_1$ is equal to

(a) n^2y (b) ny^2

(c) n^2y^2 (d) none of these

28. If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is

(a) −1 (b) 1

(c) log 2 (d) − log 2

29. The solution set of $f'(x) > g'(x)$ where $f(x) = (1/2)\,5^{2x+1}$ and $g(x) = 5^x + 4x\log_e 5$ is

(a) $(1, \infty)$ (b) $(0, 1)$

(c) $[0, \infty)$ (d) $(0, \infty)$

30. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$, then $f^2(19) + g^2(19)$ is

(a) 16 (b) 32

(c) 64 (d) none of these

Answers

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Objective Questions Type I [Only one correct answer]

1. (d)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (b)
11. (d)	**12.** (c)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (b)
21. (b)	**22.** (b)	**23.** (c)	**24.** (c)	**25.** (c)	**26.** (b)	**27.** (a)	**28.** (a)	**29.** (d)	**30.** (b)

Topics: Rolle's theorem , LMVT , Increasing/Decreasing..& Maxima/Minima

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. Let f and g be non-increasing and non-decreasing functions respectively from $[0, \infty)$ to $[0, \infty)$ and $h(x) = f(g(x))$, $h(0) = 0$, then in $[0, \infty)$, $h(x) - h(1)$ is
   (a) < 0 (b) > 0
   (c) $= 0$ (d) increasing

2. If $ax^2 + \frac{b}{x} \geq c \; \forall \; x > 0$, where $a > 0$, $b > 0$, then
   (a) $27\, ab^2 \geq 4c^3$ (b) $27\, ab^3 \leq 4c^3$
   (c) $ab^2 \geq c^3$ (d) $ab^3 \leq c^3$

3. In $[0, 1]$, Langranges mean value theorem is NOT applicable to
   (a) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$
   (b) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$
   (c) $f(x) = x|x|$
   (d) $f(x) = |x|$

4. If $f(x) = x^\alpha \ln x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in $[0, 1]$ is
   (a) -2 (b) -1
   (c) 0 (d) $1/2$

5. The function $f(x) = \sin^4 x + \cos^4 x$ increases, if
   (a) $0 < x < \frac{\pi}{8}$ (b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
   (c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

6. Let $y = x^2 e^{-x}$, then the interval in which y increases with respect to x is
   (a) $(-\infty, \infty)$ (b) $(-2, 0)$
   (c) $(2, \infty)$ (d) $(0, 2)$

7. The function $f(x) = \cos\left(\frac{\pi}{x}\right)$ is decreasing in the interval
   (a) $(2n+1, 2n)$, $n \in N$ (b) $\left(\frac{1}{2n+1}, 2n\right)$, $n \in N$
   (c) $\left(\frac{1}{2n+2}, \frac{1}{2n+1}\right)$, $n \in N$ (d) none of these

8. The set of all values of a for which the function
   $$f(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right) x^5 - 3x + \log 5$$
   decreases for all real x is
   (a) $(-\infty, \infty)$ (b) $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup (1, \infty)$
   (c) $\left(-3, 5 - \frac{\sqrt{27}}{2}\right) \cup (2, \infty)$ (d) $[1, \infty)$

9. On which of the following intervals is the function $x^{100} + \sin x - 1$ decreasing ?
   (a) $(0, \pi/2)$ (b) $(0, 1)$
   (c) $\left(\frac{\pi}{2}, \pi\right)$ (d) None of these

10. The function $f(x) = \log_e(1 + x) - \frac{2x}{2 + x}$ is increasing on
    (a) $(0, \infty)$ (b) $(-\infty, 0)$
    (c) $(-\infty, \infty)$ (d) none of these

11. The length of longest interval in which Rolle's theorem can be applied for the function
    $$f(x) = |x^2 - a^2|, \; (a > 0) \text{ is}$$
    (a) $2a$ (b) $4a^2$
    (c) $a\sqrt{2}$ (d) a

12. The points of extremum of the function $F(x) = \int_1^x e^{-t^2/2}(1-t^2)dt$ are
(a) ± 1
(b) 0
(c) $\pm 1/2$
(d) ± 2

13. A function f such that $f'(2) = f''(2) = 0$ and f has a local maximum of -17 at 2 is
(a) $(x-2)^4$
(b) $3-(x-2)^4$
(c) $-17-(x-2)^4$
(d) none of these

14. The difference between the greatest and the least value of the function
$f(x) = \int_0^x (t^2+t+1)\,dt$ on $[2, 3]$ is
(a) 37/6
(b) 47/6
(c) 57/6
(d) 59/6

15. Let $f(x) = a-(x-3)^{8/9}$, then maxima of $f(x)$ is
(a) 3
(b) $a-3$
(c) a
(d) none of these

16. Let $f(x)$ be a differential function for all x, if $f(1) = -2$ and $f'(x) \geq 2$ for all x in $[1, 6]$, then minimum value of $f(6)$ is equal to
(a) 2
(b) 4
(c) 6
(d) 8

17. The point in the interval $[0, 2\pi]$, where $f(x) = e^x \sin x$ has maximum slope is
(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{2}$
(c) π
(d) none of these

18. Let $f(x) = \begin{cases} |x^3+x^2+3x+\sin x|\left(3+\sin\frac{1}{x}\right), & x \neq 0, \\ 0 & , x = 0 \end{cases}$
then number of points (where $f(x)$ attains its minimum value) is
(a) 1
(b) 2
(c) 3
(d) infinite many

19. If $f(x) = a\log_e|x| + bx^2 + x$ has extremum at $x = 1$ and $x = 3$, then
(a) $a = -3/4, b = -1/8$
(b) $a = 3/4, b = -1/8$
(c) $a = -3/4, b = 1/8$
(d) none of these

20. Let $f(x) = 1 + 2x^2 + 2^2x^4 + \ldots + 2^{10}x^{20}$. Then $f(x)$ has
(a) more than one minimum
(b) exactly one minimum
(c) at least one maximum
(d) none of the above

21. Let $f(x) = \begin{cases} \sin^{-1}\alpha + x^2, & 0 < x < 1 \\ 2x, & x \geq 1 \end{cases}$
$f(x)$ can have a minimum at $x = 1$ is the value of α is
(a) 1
(b) -1
(c) 0
(d) none of these

22. A differentiable function $f(x)$ has a relative minimum at $x = 0$, then the function $y = f(x) + ax + b$ has a relative minimum at $x = 0$ for
(a) all a and all b
(b) all b if $a = 0$
(c) all $b > 0$
(d) all $a > 0$

23. The function
$f(x) = \int_1^x \{2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\}\,dt$
attains its maximum at x is equal to
(a) 1
(b) 2
(c) 3
(d) 4

24. Assuming that the petrol burnt in a motor boat varies as the cube of its velocity, the most economical speed, when going against a current of c km/h is
(a) $(3c/2)$ km/h
(b) $(3c/4)$ km/h
(c) $(5c/2)$ km/h
(d) $(c/2)$ km/h

25. N Characters of information are held on magnetic tape, in batches of x characters each, the batch processing time is $\alpha + \beta x^2$ seconds, α and β are constants. The optical value of x for fast processing is
(a) $\frac{\alpha}{\beta}$
(b) $\frac{\beta}{\alpha}$
(c) $\sqrt{\frac{\alpha}{\beta}}$
(d) $\sqrt{\frac{\beta}{\alpha}}$

26. The minimum value of the function defined by $f(x) = \text{maximum}\{x, x+1, 2-x\}$ is
(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) $\frac{3}{2}$

27. On $[1, e]$, the least and greatest values of $f(x) = x^2 \ln x$ is
(a) $e, 1$
(b) $1, e$
(c) $0, e^2$
(d) none of these

28. The minimum value of $\left(1+\frac{1}{\sin^n\alpha}\right)\left(1+\frac{1}{\cos^n\alpha}\right)$ is
(a) 1
(b) 2
(c) $(1+2^{n/2})^2$
(d) none of these

29. The fuel charges for running a train are proportional to the square of the speed generated in mile/h and costs Rs 48 per h at 16 miles/h. The most economical speed if the fixed charges *ie,* salaries etc amount to Rs 300 per h
(a) 10 mile/h
(b) 20 mile/h
(c) 30 mile/h
(d) 40 mile/h

30. The maximum area of the rectangle that can be inscribed in a circle of radius r is
(a) πr^2
(b) r^2
(c) $\frac{\pi r^2}{4}$
(d) $2r^2$

Answers

Objective Questions Type I [Only one correct answer]

1. (c)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (b)	**9.** (d)	**10.** (a)
11. (a)	**12.** (a)	**13.** (c)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (a)	**20.** (b)
21. (d)	**22.** (b)	**23.** (a)	**24.** (a)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (c)	**29.** (d)	**30.** (d)

Topics: Indefinite Integral & Definite Integral

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. $\int \frac{(x^2-1)}{x^3\sqrt{(2x^4-2x^2+1)}}\,dx$ is equal to

(a) $\frac{\sqrt{(2x^4-2x^2+1)}}{x(x^2-1)}+c$ (b) $\frac{\sqrt{(2x^4-2x^2+1)}}{x^3}+c$

(c) $\frac{\sqrt{(2x^4-2x^2+1)}}{x^2}+c$ (d) $\frac{\sqrt{(2x^4-2x^2+1)}}{2x^2}+c$

2. The integral $\int \frac{dx}{\left(\sqrt{x}+\sqrt[3]{x^2}\right)}$ represents the function

(a) $6\{\sqrt[3]{x^2}-\sqrt[3]{x}+\ln|1+\sqrt[3]{x}|\}+c$

(b) $3\sqrt[3]{x}-6\sqrt[6]{x}+6\ln|1+\sqrt[6]{x}|+c$

(c) $3\sqrt[3]{x}+6\sqrt[6]{x}+6\ln|1+\sqrt[6]{x}|+c$

(d) $6\sqrt[3]{x^2}-3\sqrt[3]{x}+6\ln|1+\sqrt[3]{x}|+c$

3. Given $f(x)=\begin{vmatrix} 0 & x^2-\sin x & \cos x-2 \\ \sin x-x^2 & 0 & 1-2x \\ 2-\cos x & 2x-1 & 0 \end{vmatrix}$, then

$\int f(x)\,dx$ is equal to

(a) $\frac{x^3}{3}-x^2\sin x+\sin 2x+c$

(b) $\frac{x^3}{3}-x^2\sin x-\cos 2x+c$

(c) $\frac{x^3}{3}-x^2\cos x-\cos 2x+c$

(d) none of the above

4. Let $x^2+1\neq n\pi,\ n\in N$, then

$\int x\sqrt{\left\{\frac{2\sin(x^2+1)-\sin\{2x^2+1\}}{2\sin(x^2+1)+\sin\{2x^2+1\}}\right\}}\,dx$ is

(a) $\ln\left|\frac{1}{2}\sec(x^2+1)\right|+c$ (b) $\ln\left|\sec\left\{\frac{1}{2}(x^2+1)\right\}\right|+c$

(c) $\frac{1}{2}\ln|\sec(x^2+1)|+c$ (d) $\ln|\sec(x^2+1|+c$

5. $\int|\ln x|\,dx$ equals $(0<x<1)$

(a) $x+x|\ln x|+c$ (b) $x|\ln x|-x+c$

(c) $x+|\ln x|+c$ (d) $x-|\ln x|+c$

6. $\int(x-{}^{11}C_1x^2+{}^{11}C_2x^3-{}^{11}C_3x^4+\ldots-{}^{11}C_{11}x^{12})\,dx$ equals

(a) $\frac{(1-x)^{12}}{12}-\frac{(1-x)^{11}}{11}+c$ (b) $\frac{(1-x)^{13}}{13}-\frac{(1-x)^{12}}{12}+c$

(c) $\frac{(1-x)^{11}}{11}-\frac{(1-x)^{12}}{12}+c$ (d) $\frac{(1-x)^{12}}{12}-\frac{(1-x)^{13}}{13}+c$

7. $\int\sqrt{(x-3)}\,\{\sin^{-1}(\ln x)+\cos^{-1}(\ln x)\}\,dx$ equals

(a) $\frac{\pi}{3}(x-3)^{3/2}+c$ (b) 0

(c) does not exist (d) none of these

8. $\int\frac{3+2\cos x}{(2+3\cos x)^2}\,dx$ is equal to

(a) $\left(\frac{\sin x}{2+3\cos x}\right)+c$ (b) $\left(\frac{2\cos x}{2+3\sin x}\right)+c$

(c) $\left(\frac{2\cos x}{2+3\cos x}\right)+c$ (d) $\left(\frac{2\sin x}{2+3\sin x}\right)+c$

9. $\int\sec^{4/9}\theta\,\mathrm{cosec}^{14/9}\theta\,d\theta$ is equal to

(a) $\frac{5}{9}(\tan\theta)^{-5/9}+c$ (b) $-\frac{9}{5}(\tan\theta)^{-5/9}+c$

(c) $\frac{9}{5}(\tan\theta)^{-9/5}+c$ (d) $-\frac{5}{9}(\tan\theta)^{-9/5}+c$

10. If $\int\frac{(\sqrt{x})^5}{(\sqrt{x})^7+x^6}\,dx=\lambda\ln\left(\frac{x^a}{x^a+1}\right)+c$, then $a+\lambda$ is

(a) $=2$ (b) >2

(c) <2 (d) $=1$

11. $\int|x|\ln|x|\,dx$ equals $(x\neq 0)$

(a) $\frac{x^2}{2}\ln|x|-\frac{x^2}{4}+c$

(b) $\frac{1}{2}x|x|\ln x+\frac{1}{4}x|x|+c$

(c) $-\frac{x^2}{2}\ln|x|+\frac{x^2}{4}+c$

(d) $\frac{1}{2}x|x|\ln|x|-\frac{1}{4}x|x|+c$

12. If a particle is moving with velocity $v(t)=\cos \pi t$ along a straight line such that at $t=0$, $s=4$ its position function is given by

(a) $\frac{1}{\pi}\cos \pi t+2$ (b) $-\frac{1}{\pi}\sin \pi t+4$

(c) $\frac{1}{\pi}\sin \pi t+4$ (d) none of these

13. Let $f(x)$ be a function such that, $f(0)=f'(0)=0$, $f''(x)=\sec^4 x+4$, then the function is

(a) $\ln|(\sin x)|+\frac{1}{3}\tan^3 x+x$

(b) $\frac{2}{3}\ln|(\sec x)|+\frac{1}{6}\tan^2 x+2x^2$

(c) $\ln|\cos x|+\frac{1}{6}\cos^2 x+\frac{x^2}{5}$

(d) none of the above

14. $\int \frac{(2x^{12}+5x^9)}{(x^5+x^3+1)^3}dx$ is equal to

(a) $\frac{x^2+2x}{(x^5+x^3+1)^2}+c$

(b) $\frac{x^{10}}{2(x^5+x^3+1)^2}+c$

(c) $\ln|x^5+x^3+1|+\sqrt{(2x^7+5x^4)}+c$

(d) none of the above

15. If $\int f(x)\cos x\,dx=\frac{1}{2}\{f(x)\}^2+c$, then $f(x)$ is

(a) $x+c$ (b) $\sin x+c$

(c) $\cos x+c$ (d) c

16. $\int(\sin 2x-\cos 2x)\,dx=\frac{1}{\sqrt{2}}\sin(2x-a)+b$

(a) $a=\frac{5\pi}{4}, b\in R$ (b) $a=-\frac{5\pi}{4}, b\in R$

(c) $a=\frac{\pi}{4}, b\in R$ (d) none of these

17. The primitive function of the function $f(x)=\frac{\sqrt{(a^2-x^2)}}{x^4}$ is

(a) $c+\frac{\sqrt{a^2-x^2}}{3a^2x^3}$ (b) $c-\frac{(a^2-x^2)^{3/2}}{2a^2x^2}$

(c) $c-\frac{(a^2-x^2)^{3/2}}{3a^2x^3}$ (d) none of these

18. The antiderivative of $f(x)=\frac{1}{3+5\sin x+3\cos x}$, whose graph passes through the point (0, 0) is

(a) $\frac{1}{5}\left(\ln\left|1-\frac{5}{3}\tan(x/2)\right|\right)$

(b) $\frac{1}{5}\left(\ln\left|1+\frac{5}{3}\tan(x/2)\right|\right)$

(c) $\frac{1}{5}\left(\ln\left|1+\frac{5}{3}\cot(x/2)\right|\right)$

(d) none of the above

19. $\int x^x(1+\ln|x|)\,dx$ is equal to

(a) $x^x\ln|x|+c$ (b) $e^{x^x}+c$

(c) x^x+c (d) none of these

20. If $\int\cos^4 x\,dx=Ax+B\sin 2x+C\sin 4x+D$, then $\{A,B,C\}$ equals

(a) $\left\{\frac{3}{8},\frac{1}{32},\frac{1}{4}\right\}$ (b) $\left\{\frac{3}{8},\frac{1}{4},\frac{1}{32}\right\}$

(c) $\left\{\frac{1}{32},\frac{1}{4},\frac{3}{8}\right\}$ (d) $\left\{\frac{1}{4},\frac{3}{8},\frac{1}{32}\right\}$

21. $\int\frac{1}{x^2(x^4+1)^{3/4}}dx$ is equal to

(a) $\left(1+\frac{1}{x^4}\right)^{1/4}+c$ (b) $(x^4+1)^{1/4}+c$

(c) $\left(1-\frac{1}{x^4}\right)^{1/4}+c$ (d) $-\left(1+\frac{1}{x^4}\right)^{1/4}+c$

22. Let the equation of a curve passing through the point (0, 1) be given by $y=\int x^2\cdot e^{x^3}dx$. If the equation of the curve is written in the form $x=f(y)$, then $f(y)$ is

(a) $\sqrt{\ln\left|\frac{3y-2}{3}\right|}$ (b) $\sqrt[3]{\ln\left|\frac{2-3y}{3}\right|}$

(c) $\sqrt[3]{\ln\left|\frac{3y-2}{3}\right|}$ (d) none of these

23. $\int\frac{xe^x}{(1+x)^2}dx$ is equal to

(a) $\frac{e^x}{x+1}+c$ (b) $e^x(x+1)+c$

(c) $-\frac{e^x}{(x+1)^2}+c$ (d) $\frac{e^x}{1+x^2}+c$

24. If the derivative of $f(x)$ w.r.t. x is $\frac{(1/2)-\sin^2 x}{f(x)}$, then $f(x)$ is a periodic function with period

(a) $\pi/2$ (b) π

(c) 2π (d) not defined

25. $\int x^{-2/3}(1+x^{1/2})^{-5/3}dx$ is equal to

(a) $3(1+x^{-1/2})^{-1/3}+c$ (b) $3(1+x^{-1/2})^{-2/3}+c$

(c) $3(1+x^{1/2})^{-2/3}+c$ (d) none of these

26. $\int\frac{\cos 4x-1}{\cot x-\tan x}dx$ is equal to

(a) $-\frac{1}{2}\cos 4x+c$ (b) $-\frac{1}{4}\cos 4x+c$

(c) $-\frac{1}{2}\sin 2x+c$ (d) none of these

27. If $f(x) = \lim_{n \to \infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}, x > 1$, then

$\int \frac{xf(x) \ln (x + \sqrt{(1 + x^2)})}{\sqrt{(1 + x^2)}} dx$ is

(a) $\ln (x + \sqrt{(1 + x^2)}) - x + c$

(b) $\frac{1}{2}\{x^2 \ln (x + \sqrt{(1 + x^2)}) - x^2\} + c$

(c) $x \ln (x + \sqrt{(1 + x^2)}) - \ln (x + \sqrt{(1 + x^2)}) + c$

(d) none of the above

28. The value of the integral $\int \frac{dx}{x^n (1 + x^n)^{1/n}}, n \in N$ is

(a) $\frac{1}{(1-n)}\left(1 + \frac{1}{x^n}\right)^{1-\frac{1}{n}} + c$

(b) $\frac{1}{(1+n)}\left(1 - \frac{1}{x^n}\right)^{1+\frac{1}{n}} + c$

(c) $-\frac{1}{(1-n)}\left(1 - \frac{1}{x^n}\right)^{1-\frac{1}{n}} + c$

(d) $-\frac{1}{(1+n)}\left(1 + \frac{1}{x^n}\right)^{1+\frac{1}{n}} + c$

29. The value of the integral $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$ is

(a) $\sin x - 6 \tan^{-1}(\sin x) + c$

(b) $\sin x - 2 (\sin x)^{-1} + c$

(c) $\sin x - 2 (\sin x)^{-1} - 6 \tan^{-1}(\sin x) + c$

(d) $\sin x - 2 (\sin x)^{-1} + 5 \tan^{-1}(\sin x) + c$

30. If $\int f(x) \sin x \cos x \, dx = \frac{1}{2(b^2 - a^2)} \ln f(x) + c$, then $f(x)$ is equal to

(a) $\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$

(b) $\frac{1}{a^2 \sin^2 x - b^2 \cos^2 x}$

(c) $\frac{1}{a^2 \cos^2 x + b^2 \sin^2 x}$

(d) none of these

31. If l^r means ln ln ln ... x, the ln being repeated r times, then $\int \{x\, l(x)\, l^2(x)\, l^3(x) \ldots l^r(x)\}^{-1} dx$ is equal to

(a) $l^{r+1}(x) + c$

(b) $\frac{l^{r+1}(x)}{r+1} + c$

(c) $l^r(x) + c$

(d) none of these

32. The value of $\int_0^{2\pi} \frac{dx}{e^{\sin x}+1}$ is

(a) π (b) 0
(c) 2π (d) $\pi/2$

33. $\int_0^{\pi/4} \sin x \, d(x-[x])$ is equal to (where [·] denotes the greatest integer function)

(a) 1/2 (b) $1-\frac{1}{\sqrt{2}}$
(c) 1 (d) none of these

34. The value of the integral $I=\int_1^{\infty} \frac{(x^2-2)}{x^3\sqrt{(x^2-1)}} dx$ is

(a) 0 (b) 2/3
(c) 4/3 (d) none of these

35. If $I_1=\int_0^{3\pi} f(\cos^2 x)\,dx$ and $I_2=\int_0^{\pi} f(\cos^2 x)\,dx$, then

(a) $I_1=I_2$ (b) $I_1=2I_2$
(c) $I_1=5I_2$ (d) none of these

36. If $I=\int_0^{50\pi} \sqrt{(1-\cos 2x)}\,dx$, then the value of I is

(a) $50\sqrt{2}$ (b) $100\sqrt{2}$
(c) $25\sqrt{2}$ (d) none of these

37. Let $f(x)$ be an odd function in the interval $\left[-\frac{T}{2},\frac{T}{2}\right]$, with a period T. Then $F(x)=\int_a^x f(t)\,dt$ is

(a) periodic with period T
(b) non periodic
(c) periodic with period $2T$
(d) periodic with period aT

38. If $\int_0^{10} f(x)\,dx=5$, then $\sum_{k=1}^{10}\int_0^1 f(k-1+x)\,dx$ is

(a) 50 (b) 10
(c) 5 (d) none of these

39. The value of the integral $\left|\int_0^{2\pi}[2\sin x]\,dx\right|$ is ([·] denotes the greatest integer function)

(a) π (b) 2π
(c) 3π (d) 4π

40. Let $f(x)=$ minimum $\left(|x|, 1-|x|, \frac{1}{4}\right)$, $\forall x\in R$, then the value of $\int_{-1}^{1} f(x)\,dx$ is equal to

(a) $\frac{1}{32}$ (b) $\frac{3}{8}$
(c) $\frac{3}{32}$ (d) none of these

41. If $\int_{1/2}^{2}\frac{1}{x}\operatorname{cosec}^{101}\left(x-\frac{1}{x}\right)dx=k$, then the value of k is

(a) 1 (b) 1/2
(c) 0 (d) 1/101

42. If $\int_0^{\infty} e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$ and $\int_0^{\infty} e^{-ax^2}dx$, $a>0$ is

(a) $\frac{\sqrt{\pi}}{2}$ (b) $\frac{\sqrt{\pi}}{2a}$
(c) $2\frac{\sqrt{\pi}}{a}$ (d) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$

43. If $\int_0^1 e^{x^2}(x-\alpha)\,dx=0$, then

(a) $1<\alpha<2$ (b) $\alpha<0$
(c) $0<\alpha<1$ (d) $\alpha=0$

44. If $[x]$ denotes the greatest integer less than or equal to x, then $\int_0^{\infty}\left[\frac{2}{e^x}\right]dx$ is equal to

(a) $\log_e 2$ (b) e^2
(c) 0 (d) $\frac{2}{e}$

45. If $\int_0^x f(t)\,dt=x+\int_x^1 t\,f(t)\,dt$, then the value of $f(1)$ is

(a) 1/2 (b) 0
(c) 1 (d) −1/2

46. The value of $\int_{\pi/4}^{3\pi/4}\frac{x}{1+\sin x}dx$ is equal to

(a) $(\sqrt{2}-1)\pi$ (b) $(\sqrt{2}+1)\pi$
(c) π (d) none of these

47. Let f be a positive function. If

$$I_1=\int_{1-k}^{k} x\,f\{x(1-x)\}\,dx,$$

$$I_2=\int_{1-k}^{k} f\{x(1-x)\}\,dx$$

where $2k-1>0$, then $I_1:I_2$ is equal to

(a) 2 : 1 (b) $k:1$
(c) 1 : 2 (d) 1 : 1

48. Let $f(x)=\frac{1}{2}a_0+\sum_{i=1}^{n} a_i\cos(ix)+\sum_{j=1}^{n} b_j\sin(jx)$, then $\int_{-\pi}^{\pi} f(x)\cos kx\,dx$ is equal to

(a) a_k (b) b_k
(c) πa_k (d) πb_k

49. The value of the definite integral $\int_0^1\frac{x\,dx}{(x^3+16)}$ lies in the interval $[a,b]$. Then smallest such interval is

(a) $\left[0,\frac{1}{17}\right]$ (b) [0, 1]
(c) $\left[0,\frac{1}{27}\right]$ (d) none of these

50. Value of $\int_2^3\frac{dx}{\sqrt{(1+x^3)}}$ is

(a) less than 1 (b) greater than 2
(c) lies between 3 and 4 (d) none of these

51. Suppose for every integer n, $\int_n^{n+1} f(x)\,dx=n^2$, the value of $\int_{-2}^{4} f(x)\,dx$ is

(a) 16 (b) 14
(c) 19 (d) none of these

52. The value of $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\, dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\, dt$ is

(a) $\pi/2$ (b) 1
(c) $\pi/4$ (d) none of these

53. If $I = \int_0^1 \cos\left(2\cot^{-1}\sqrt{\left(\frac{1-x}{1+x}\right)}\right) dx$, then

(a) $I > \frac{1}{2}$ (b) $I = -\frac{1}{2}$
(c) $0 < I < \frac{1}{2}$ (d) none of these

54. The value of $\int_0^2 [x^2 - 1]\, dx$, where $[x]$ denotes the greatest integer function, is given by

(a) $3 - \sqrt{3} - \sqrt{2}$ (d) 2
(c) 1 (d) none of these

55. Let $\int_0^x \left(\frac{bt\cos 4t - a\sin 4t}{t^2}\right) dt = \frac{a\sin 4x}{x}$, then a and b are given by

(a) $a = 1/4, b = 1$ (b) $a = 2, b = 2$
(c) $a = -1, b = 4$ (d) $a = 2, b = 4$

56. If $f(x) = \cos x - \int_0^x (x - t) f(t)\, dt$, then $f''(x) + f(x)$ equals

(a) $-\cos x$ (b) 0
(c) $\int_0^x (x-t) f(t)\, dt$ (d) $-\int_0^{-x} (x-t) f(t)\, dt$

57. Let $f(x) = \max\{x + |x|, x - [x]\}$, where $[x]$ denotes the greatest integer $\le x$. Then $\int_{-2}^{2} f(x)\, dx$ is equal to

(a) 3 (b) 2
(c) 1 (d) none of these

58. The value of $\int_{-2}^{2} \frac{\sin^2 x}{[x/\pi] + [1/2]} dx$, where $[x]$ denotes the greatest integer $\le x$, is

(a) 1 (b) 0
(c) $4 - \sin 4$ (d) none of these

59. The value of $\int_{-1}^{1} \max\{2 - x, 2, 1 + x\}\, dx$ is

(a) 4 (b) 9/2
(c) 2 (d) none of these

60. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g be the function satisfying $f(x) + g(x) = x^2$ the value of the integral $\int_0^1 f(x) g(x)\, dx$ is

(a) $\frac{1}{4}(e - 7)$ (b) $\frac{1}{4}(e - 2)$
(c) $\frac{1}{2}(e - 3)$ (d) none of these

61. The value of $\int_0^{\pi} \left(\sum_{r=0}^{3} a_r \cos^{3-r} x \sin^r x\right) dx$ depends on

(a) a_0 and a_2 (b) a_1 and a_2
(c) a_0 and a_3 (d) a_1 and a_3

62. Let $f : R \to R$ such that $f(x + 2y) = f(x) + f(2y) + 4xy$ $\forall\, x, y \in R$ and $f'(0) = 0$. If $I_1 = \int_0^1 f(x)\, dx$, $I_2 = \int_{-1}^{0} f(x)\, dx$ and $I_3 = \int_{1/2}^{2} f(x)\, dx$, then

(a) $I_1 = I_2 > I_3$ (b) $I_1 > I_2 > I_3$
(c) $I_1 = I_2 < I_3$ (d) $I_1 < I_2 < I_3$

63. If $\int_a^b \frac{x^n}{x^n + (16 - x)^n} dx = 6$, then

(a) $a = 4, b = 12, n \in R$ (b) $a = 2, b = 14, n \in R$
(c) $a = -4, b = 20, n \in R$ (d) $a = 2, b = 8, n \in R$

64. If $f(x)$ and $g(x)$ are continuous functions, then

$$\int_{\ln \lambda}^{\ln(1/\lambda)} \frac{f\left(\frac{x^2}{4}\right)(f(x) - f(-x))}{g\left(\frac{x^2}{4}\right)(g(x) + g(-x))} dx \text{ is}$$

(a) depend on λ (b) a non-zero constant
(c) zero (d) none of these

65. The value of $\int_0^{\pi/2} \frac{1 + 2\cos x}{(2 + \cos x)^2} dx$ is

(a) $-\frac{1}{2}$ (b) 2
(c) $\frac{1}{2}$ (d) none of these

66. The value of $\int_0^{2\pi} \frac{x\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$
(c) π^2 (d) $2\pi^2$

67. Consider the integrals

$$I_1 = \int_0^1 e^{-x}\cos^2 x\, dx,\ I_2 = \int_0^1 e^{-x^2}\cos^2 x\, dx,$$
$$I_3 = \int_0^1 e^{-x^2} dx \text{ and } I_4 = \int_0^1 e^{-x^2/2} dx$$

let I be the greatest integral among I_1, I_2, I_3, I_4, then

(a) $I = I_1$ (b) $I = I_2$
(c) $I = I_3$ (d) $I = I_4$

68. If $f(x) = \int_2^{x^2} \frac{(\sin^{-1}\sqrt{t})^2}{\sqrt{t}} dt$, then the value of $(1 - x^2)\{f''(x)\}^2 - 2f'(x)$ at $x = \frac{1}{\sqrt{2}}$ is

(a) $2 - \pi$ (b) $3 + \pi$
(c) $4 - \pi$ (d) none of these

69. If for $x \ne 0$, $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \ne b$, then $\int_1^2 xf(x)\, dx$ is equal to

(a) $\frac{b - 9a}{9(a^2 - b^2)}$ (b) $\frac{b - 9a}{b(a^2 - b^2)}$
(c) $\frac{b - 9a}{6(a^2 - b^2)}$ (d) none of these

70. Let f and g be two continuous functions. Then $\int_{-\pi/2}^{\pi/2}\{f(x)+f(-x)\}\{g(x)-g(-x)\}\,dx$ is equal to

(a) π (b) 1
(c) -1 (d) 0

71. Let $f(x)$ be a continuous function such that $f(a-x)+f(x)=0$ for all $x\in[0,a]$, then $\int_0^a \frac{dx}{1+e^{f(x)}}$ is equal to

(a) a (b) $\frac{a}{2}$
(c) $f(a)$ (d) $\frac{1}{2}f(a)$

72. The equation

$$\int_{-\pi/4}^{\pi/4}\left(a|\sin x|+\frac{b\sin x}{1+\cos x}+c\right)dx=0,$$

where a, b, c are constants, gives a relation between

(a) a, b and c (b) a and c
(c) a and b (d) b and c

73. The value of $\int_0^{16\pi/3}|\sin x|\,dx$ is

(a) 17/2 (b) 19/2
(c) 21/2 (d) none of these

74. The value of $\int_0^{1000} e^{x-[x]}\,dx$, is ([·] denotes the greatest integer function)

(a) $1000e$ (b) $1000(e-1)$
(c) $1001(e-1)$ (d) none of these

75. If $f'''(x)=k$ in $[0,a]$, then

$$\int_0^a f(x)\,dx-\left\{x f(x)-\frac{x^2}{2!}f'(x)+\frac{x^3}{3!}f''(x)\right\}_0^a \text{ is}$$

(a) $-ka^4/12$ (b) $ka^4/24$
(c) $-ka^4/24$ (d) none of these

76. The value of $\int_0^{n^2}[\sqrt{x}]\,dx$, ([·] denotes the greatest integer function), $n\in N$, is

(a) $\left\{\frac{n(n+1)}{2}\right\}^2$ (b) $\frac{1}{6}n(n-1)(4n+1)$
(c) Σn^2 (d) $\frac{n(n+1)(n+2)}{6}$

77. $\lim_{n\to\infty}\frac{(n!)^{1/n}}{n}$ equals

(a) e (b) e^{-1}
(c) 1 (d) none of these

78. If [.] stands for the greatest integer function, the value of $\int_4^{10}\frac{[x^2]\,dx}{[x^2-28x+196]+[x^2]}$ is

(a) 0 (b) 1
(c) 3 (d) none of these

79. $\int_{\cos\cos^{-1}\alpha}^{\sin\sin^{-1}\beta}\left|\frac{\cos(\cos^{-1}x)}{\sin(\sin^{-1}x)}\right|dx$ is equal to

(a) 1 (b) 0
(c) $\beta-\alpha$ (d) none of these

80. If $na=1$ always and $n\to\infty$, then the value of $\Pi\{1+(ra)^2\}^{1/r}$ is

(a) 1 (b) $e^{\pi^2/8}$
(c) $e^{\pi^2/24}$ (d) $e^{-\pi^2/12}$

81. The value of $\int_0^{100}[\tan^{-1}x]\,dx$ is (where [·] denotes the greatest integer function)

(a) 100 (b) $100-\tan^{-1}1$
(c) $100-\tan 1$ (d) none of these

82. If $f(x)$ satisfies the requirements of Rolle's theorem in [1, 2] and $f'(x)$ is continuous in [1, 2], then $\int_1^2 f'(x)\,dx$ is equal to

(a) 0 (b) 1
(c) 3 (d) -1

83. The value of $\int_{-2}^{2}\min(x-[x],-x-[-x])\,dx$ is ([·] denotes the greatest integer function)

(a) 0 (b) 1
(c) 2 (d) none of these

84. The value of $\int_0^2[x^2-x+1]\,dx$, (where [·] denotes the greatest integer function) is given by

(a) $\frac{5-\sqrt{5}}{2}$ (b) $\frac{6-\sqrt{5}}{2}$
(c) $\frac{7-\sqrt{5}}{2}$ (d) $\frac{8-\sqrt{5}}{2}$

85. The value of $\lim_{n\to\infty}\left(\frac{1^k+2^k+\ldots+n^k}{n^{k+1}}\right)$ is

(a) $\frac{1}{k+2}$ (b) $\frac{1}{k+1}$
(c) $\frac{2}{k+3}$ (d) 0

86. The value of the integral $\int_{1/e}^{e}|\ln x|\,dx$ is

(a) $1-1/e$ (b) $2(1-1/e)$
(c) $e^{-1}-1$ (d) none of these

87. $f(x)$ is continuous periodic function with period T, then the integral $I=\int_a^{a+T}f(x)\,dx$ is

(a) equal to $2a$ (b) equal to $3a$
(c) independent of a (d) none of these

88. $\lim_{n\to\infty}\frac{1}{n}\sum_{r=n+1}^{2n}\ln\left(1+\frac{r}{n}\right)$ equals

(a) $\ln\left(\frac{27}{4e}\right)$ (b) $\ln\left(\frac{27}{e^2}\right)$
(c) $\ln\left(\frac{4}{e}\right)$ (d) none of these

89. Given that n is odd and m is even integer. The value of $\int_0^{\pi} \cos mx \sin nx \, dx$ is

(a) $\frac{2m}{n^2 - m^2}$ (b) $\frac{2n}{n^2 - m^2}$

(c) $\frac{m^2 + n^2}{n^2 - m^2}$ (d) none of these

90. $\int_{a/4}^{3a/4} \frac{\sqrt{x}}{\sqrt{(a-x)} + \sqrt{x}} dx$ is equal to

(a) $\frac{a}{2}$ (b) a

(c) $-a$ (d) none of these

91. The value of $\int_0^2 \left|\cos\left(\frac{\pi x}{2}\right)\right| dx$ is

(a) 2π (b) $\pi/2$

(c) $3/4\pi$ (d) $4/\pi$

92. If $f'(x) = f(x) + \int_0^1 f(x)\, dx$ and given $f(0) = 1$, then $f(x)$ is equal to

(a) $\frac{e^x}{2-e} + \left(\frac{1+e}{1-e}\right)$

(b) $\frac{2e^x}{3-e} + \left(\frac{1-e}{3-e}\right)$

(c) $\frac{e^x}{2-e}$

(d) $\frac{2e^x}{3-e}$

93. The value of the integral $\int_0^1 \frac{x^{\alpha} - 1}{\ln x} dx$, is

(a) $\ln \alpha$ (b) $2 \ln (\alpha + 1)$

(c) $3 \ln \alpha$ (d) none of these

94. The value of $\int_{\pi}^{2\pi} [2 \sin x]\, dx$, (where [·] represents the greatest integer function) is

(a) $-\frac{5\pi}{3}$ (b) $-\pi$

(c) $\frac{5\pi}{3}$ (d) -2π

95. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k^{1/a}\left\{n^{a-\frac{1}{a}} + k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$ is equal to

(a) 1 (b) 2

(c) 3 (d) none of these

96. Let $f(x) = \min(\{x\}, \{-x\}) \; \forall \; x \in R$, where $\{x\}$ denotes the fractional part of x, then $\int_{-100}^{100} f(x)\, dx$ is equal to

(a) 50 (b) 100

(c) 200 (d) none of these

97. The value of $\int_{-\pi/2}^{199\pi/2} \sqrt{(1 + \cos 2x)}\, dx$ is

(a) $50\sqrt{2}$ (b) $100\sqrt{2}$

(c) $150\sqrt{2}$ (d) $200\sqrt{2}$

98. The value of $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x)\, d(x - [x])$, (where [·] denotes the greatest integer function) is

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$

(c) $\frac{2}{n-1}$ (d) none of these

99. If $\int_{-1}^{4} f(x)\, dx = 4$ and $\int_2^4 (3 - f(x))\, dx = 7$, the value of $\int_2^{-1} f(x)\, dx$ is

(a) 2 (b) –3

(c) –5 (d) none of these

100. If $I_1 = \int_0^x e^{zx} e^{-z^2} dz$ and $I_2 = \int_0^x e^{-z^2/4} dz$, then

(a) $I_1 = e^x I_2$ (b) $I_1 = e^{x^2} I_2$

(c) $I_1 = e^{x^2/2} I_2$ (d) none of these

101. If $z = x + 3i$, then the value of $\int_2^4 \left[\arg\left|\frac{z-i}{z+i}\right|\right] dx$, (where [·] denotes the greatest integer function and $i = \sqrt{-1}$, is

(a) $3\sqrt{2}$ (b) $6\sqrt{3}$

(c) $\sqrt{6}$ (d) none of these

Answers

Objective Questions Type I [Only one correct answer]

1. (d)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (b)
11. (d)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (b)
21. (d)	**22.** (d)	**23.** (a)	**24.** (b)	**25.** (b)	**26.** (d)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (a)
31. (a)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (d)	**36.** (b)	**37.** (a)	**38.** (c)	**39.** (a)	**40.** (b)
41. (c)	**42.** (d)	**43.** (c)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (c)	**48.** (c)	**49.** (a)	**50.** (a)
51. (c)	**52.** (c)	**53.** (b)	**54.** (a)	**55.** (a)	**56.** (a)	**57.** (d)	**58.** (b)	**59.** (b)	**60.** (d)
61. (d)	**62.** (c)	**63.** (b)	**64.** (c)	**65.** (c)	**66.** (c)	**67.** (d)	**68.** (d)	**69.** (d)	**70.** (d)
71. (b)	**72.** (b)	**73.** (c)	**74.** (b)	**75.** (c)	**76.** (b)	**77.** (b)	**78.** (c)	**79.** (c)	**80.** (c)
81. (c)	**82.** (a)	**83.** (b)	**84.** (a)	**85.** (b)	**86.** (b)	**87.** (c)	**88.** (a)	**89.** (b)	**90.** (d)
91. (d)	**92.** (b)	**93.** (d)	**94.** (a)	**95.** (a)	**96.** (a)	**97.** (d)	**98.** (a)	**99.** (c)	**100.** (d)
101. (d)									

Topic: Area Under Curve

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. The area bounded by the curve $f(x) = x + \sin x$ and its inverse between the ordinates $x = 0$ to $x = 2\pi$ is
(a) 4 sq unit (b) 8 sq unit
(c) 4π sq unit (d) 8π sq unit

2. The area bounded by $y = \dfrac{\sin x}{x}$, x-axis and the ordinates $x = 0, x = \dfrac{\pi}{4}$ is
(a) $= \dfrac{\pi}{4}$ (b) $< \dfrac{\pi}{4}$
(c) $> \dfrac{\pi}{4}$ (d) $< \int_0^{\pi/4} \dfrac{\tan x}{x} dx$

3. Area of the region bounded by $[x]^2 = [y]^2$, if $x \in [1, 5]$, where [.] denotes the greatest integer function, is
(a) 4 sq unit (b) 8 sq unit
(c) 5 sq unit (d) 10 sq unit

4. If A_n is the area bounded by $y = (1 - x^2)^n$ and coordinate axes, $n \in N$, then
(a) $A_n = A_{n-1}$ (b) $A_n < A_{n-1}$
(c) $A_n > A_{n-1}$ (d) $A_n = 2A_{n-1}$

5. The area bounded by $\min(|x|, |y|) = 2$ and $\max(|x|, |y|) = 4$ is
(a) 8 sq unit (b) 16 sq unit
(c) 24 sq unit (d) 32 sq unit

6. Let $f(x) = 2\sqrt{x}$ and $g(x) = 2\sqrt{(1-x)}$ be two functions and let $f_1(x) = \max\{f(t), 0 \le t \le x, 0 \le x \le 1\}$ and $g_1(x) = \min\{g(t), 0 \le t \le x, 0 \le x \le 1\}$. Then the area bounded by $f_1(x) \le 0, g_1(x) \le 0$ and x-axis is
(a) $\dfrac{1}{3\sqrt{2}}$ sq unit (b) $\dfrac{2}{3\sqrt{2}}$ sq unit
(c) $\dfrac{1}{\sqrt{2}}$ sq unit (d) $\dfrac{4}{3\sqrt{2}}$ sq unit

7. Area bounded by the curve $y = \sqrt{(\sin[x] + [\sin x])}$, where [.] denotes the greatest integer function, lines $x = 1$ and $x = \dfrac{\pi}{2}$ and the x-axis is
(a) $\left(\dfrac{\pi}{2} - 1\right)$ sq unit (b) $\sqrt{\sin 1}\left(\dfrac{\pi}{2} - 1\right)$ sq unit
(c) $\sqrt{\cos 1}\left(\dfrac{\pi}{2} - 1\right)$ sq unit (d) $\sqrt{\dfrac{\pi}{2}}\left(\dfrac{\pi}{2} - 1\right)$ sq unit

8. The area bounded by the curves $y = \sin^{-1}|\sin x|$ and $y = (\sin^{-1}|\sin x|)^2$, $0 \le x \le 2\pi$ is
(a) $\left(\dfrac{\pi^3}{3} + \dfrac{4}{3}\right)$ sq unit (b) $\left(\dfrac{\pi^3}{6} - \dfrac{\pi^2}{2} + \dfrac{4}{3}\right)$ sq unit
(c) $\left(\dfrac{\pi^2}{2} - \dfrac{4}{3}\right)$ sq unit (d) $\left(\dfrac{\pi^2}{6} - \dfrac{\pi}{4} + \dfrac{4}{3}\right)$ sq unit

9. Area enclosed by the curve $|x + y - 1| + |2x + y - 1| = 1$ is
(a) 2 sq unit (b) 3 sq unit
(c) 6 sq unit (d) 7 sq unit

10. Area of the region bounded by the curves $y|y| \pm x|x| = 1$ and $y = |x|$ is
(a) $\dfrac{\pi}{8}$ sq unit (b) $\dfrac{\pi}{4}$ sq unit
(c) $\dfrac{\pi}{2}$ sq unit (d) π sq unit

11. The area between the curve $y = 2x^4 - x^2$, the x-axis and the ordinates of two minima of the curve is
(a) $\dfrac{7}{120}$ sq unit (b) $\dfrac{9}{120}$ sq unit
(c) $\dfrac{11}{120}$ sq unit (d) $\dfrac{13}{120}$ sq unit

12. The area bounded by the x-axis, the curve $y = f(x)$ and the lines $x = 1$ and $x = b$ is equal to $(\sqrt{(b^2+1)} - \sqrt{2})$ for all $b > 1$, then $f(x)$ is
(a) $\sqrt{(x-1)}$ (b) $\sqrt{(x+1)}$
(c) $\sqrt{(x^2+1)}$ (d) $\dfrac{x}{\sqrt{(1+x^2)}}$

13. The area of the region bounded by $1-y^2=|x|$ and $|x|+|y|=1$ is
(a) 1/3 sq unit (b) 2/3 sq unit
(c) 4/3 sq unit (d) 1 sq unit

14. Area bounded by the curve $y=(x-1)(x-2)(x-3)$ and x-axis lying between the ordinates $x=0$ and $x=3$ is equal to
(a) $\frac{9}{4}$ sq unit (b) $\frac{11}{4}$ sq unit
(c) $\frac{13}{4}$ sq unit (d) $\frac{15}{4}$ sq unit

15. The area of the figure bounded by the curves $y=|x-1|$ and $y=3-|x|$ is
(a) 2 sq unit (b) 3 sq unit
(c) 4 sq unit (d) 1 sq unit

16. Area bounded by the curve $y=x\sin x$ and x-axis between $x=0$ and $x=2\pi$ is
(a) 2π sq unit (b) 3π sq unit
(c) 4π sq unit (d) 5π sq unit

17. Let $f(x)=\min\{x+1, \sqrt{(1-x)}\}$, then area bounded by $f(x)$ and x-axis is
(a) $\frac{1}{6}$ sq unit (b) $\frac{5}{6}$ sq unit
(c) $\frac{7}{6}$ sq unit (d) $\frac{11}{6}$ sq unit

18. The area bounded by the graph $y=|[x-3]|$, the x-axis and the lines $x=-2$ and $x=3$ is ([.] denotes the greatest integer function)
(a) 7 sq unit (b) 15 sq unit
(c) 21 sq unit (d) 28 sq unit

19. The value of c for which the area of the figure bounded by the curve $y=8x^2-x^5$, the straight lines $x=1$ and $x=c$ and the x-axis is equal to $\frac{16}{3}$ is
(a) 2 (b) $\sqrt{8-\sqrt{17}}$
(c) 3 (d) -1

20. Area bounded by the curves $y=\left[\frac{x^2}{64}+2\right]$, $y=x-1$ and $x=0$ above x-axis is ([.] denotes the greatest integer function)
(a) 2 sq unit (b) 3 sq unit
(c) 4 sq unit (d) none of these

21. The slope of the tangent to a curve $y=f(x)$ at $(x, f(x))$ is $2x+1$. If the curve passes through the point (1, 2), then the area of the region bounded by the curve, the x-axis and the line $x=1$ is
(a) $\frac{5}{6}$ sq unit (b) $\frac{6}{5}$ sq unit
(c) $\frac{1}{6}$ sq unit (d) 6 sq unit

22. The area of the region bounded by the curve $a^4y^2=(2a-x)x^5$ is to that of the circle whose radius is a, is given by the ratio
(a) 4 : 5 (b) 5 : 8
(c) 2 : 3 (d) 3 : 2

23. The area bounded by $y=xe^{|x|}$ and lines $|x|=1$, $y=0$ is
(a) 4 sq unit (b) 6 sq unit
(c) 1 sq unit (d) 2 sq unit

24. The area of the figure bounded by $f(x)=\sin x$, $g(x)=\cos x$ in the first quadrant is
(a) $2(\sqrt{2}-1)$ sq unit (b) $(\sqrt{3}+1)$ sq unit
(c) $2(\sqrt{3}-1)$ sq unit (d) none of these

25. The area of the figure bounded by two branches of the curve $(y-x)^2=x^3$ and the straight line $x=1$ is
(a) $\frac{1}{3}$ sq unit (b) $\frac{4}{5}$ sq unit
(c) $\frac{5}{4}$ sq unit (d) 3 sq unit

26. If the area bounded by the x-axis, the curve $y=f(x)$ and the lines $x=c$ and $x=d$ is independent of d, $\forall\, d>c$ (c is a constant), then f is
(a) the identity function
(b) the zero function
(c) a non-zero constant function
(d) none of the above

27. The area bounded by the curves $y=|x|-1$ and $y=-|x|+1$ is
(a) 1 sq unit (b) 2 sq unit
(c) $2\sqrt{2}$ sq unit (d) $4\sqrt{2}$ sq unit

28. The area bounded by $y=x^2$, $y=[x+1]$, $0\le x<1$ and the y-axis is ([.] denotes the greatest integer function)
(a) 1/3 sq unit (b) 2/3 sq unit
(c) 1 sq unit (d) 7/3 sq unit

29. Let $f(x)$ be a continuous function such that the area bounded by the curve $y=f(x)$, the x-axis and the two ordinates $x=0$ and $x=a$ is $\left(\frac{a^2}{2}+\frac{a}{2}\sin a+\frac{\pi}{2}\cos a\right)$ sq unit, then $f\left(\frac{\pi}{2}\right)$ is
(a) $\frac{1}{2}$ (b) $\frac{\pi^2}{8}+\frac{\pi}{4}$
(c) $\frac{\pi+1}{2}$ (d) none of these

30. The area bounded by the curve $y=x^4-2x^3+x^2+3$, the axis of abscissas and two ordinates corresponding to the points of minimum of the function $y(x)$ is
(a) $\frac{10}{3}$ sq unit (b) $\frac{27}{10}$ sq unit
(c) $\frac{21}{10}$ sq unit (d) none of these

31. The area bounded by the curves $y=\ln x$, $y=\ln|x|$, $y=|\ln x|$ and $y=|\ln|x||$ is
(a) 5 sq unit (b) 2 sq unit
(c) 4 sq unit (d) none of these

Answers

Objective Questions Type I [Only one correct answer]

1. (b)	2. (b)	3. (b)	4. (b)	5. (b)	6. (d)	7. (b)	8. (b)	9. (a)	10. (b)
11. (a)	12. (d)	13. (b)	14. (b)	15. (c)	16. (c)	17. (c)	18. (b)	19. (d)	20. (c)
21. (a)	22. (b)	23. (d)	24. (a)	25. (b)	26. (b)	27. (b)	28. (b)	29. (a)	30. (d)
31. (c)									

Topic: Differential Equation

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 \cdot e^{x+c_5}$, where c_1, c_2, c_3, c_4, and c_5 are arbitrary constants, is
   (a) 2 (b) 3
   (c) 4 (d) 5

2. If $y = c_1 e^{2x} + c_2 e^{x} + c_3 e^{-x}$ satisfies the differential equation $\frac{d^3y}{dx^3} + a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$, then $\frac{a^3 + b^3 + c^3}{abc}$ is equal to
   (a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$
   (c) 0 (d) $\frac{1}{2}$

3. The order and degree of the differential equation whose general solution is given by $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{50} = \ln\left(\frac{d^2y}{dx^2}\right)$ respectively are
   (a) 2, 1 (b) 2, 50
   (c) 50, 2 (d) none of these

4. The degree and order of the differential equation of all parabolas, whose axis is x-axis are respectively
   (a) 1, 2 (b) 2, 1
   (c) 3, 2 (d) 2, 3

5. Solution of the differential equation $\frac{dy}{dx} = \frac{y\,(x - y\ln y)}{x\,(x\ln x - y)}$ is
   (a) $\frac{x\ln x + y\ln y}{xy} = c$ (b) $\frac{x\ln x - y\ln y}{xy} = c$
   (c) $\frac{\ln x}{x} + \frac{\ln y}{y} = c$ (d) $\frac{\ln x}{x} - \frac{\ln y}{y} = c$
   (where c is arbitrary constant)

6. The solution of the differential equation $\{y\,(1 + x^{-1}) + \sin y\}\,dx + (x + \ln x + x\cos y)\,dy = 0$ is
   (a) $x + y\ln x + y\sin x = c$
   (b) $xy + y\ln x + x\sin y = c$
   (c) $y + x\ln x + x\sin y = c$
   (d) $xy + x\ln x + y\sin y = c$
   (where c is arbitrary constant)

7. Solution of differential equation $(x\cos x - \sin x)\,dx = \frac{x}{y}\sin x\,dy$ is
   (a) $\sin x = \ln|xy| + c$ (b) $\ln\left|\frac{\sin x}{x}\right| = y + c$
   (c) $\left|\frac{\sin x}{xy}\right| = c$ (d) none of these
   (where c is arbitrary constant)

8. Solution of $\left(\frac{x + y - 1}{x + y - 2}\right)\frac{dy}{dx} = \left(\frac{x + y + 1}{x + y + 2}\right)$, given that $y = 1$ when $x = 1$ is
   (a) $\ln\left|\frac{(x - y)^2 - 2}{2}\right| = 2(x + y)$
   (b) $\ln\left|\frac{(x + y)^2 - 2}{2}\right| = 2(x - y)$
   (c) $\ln\left|\frac{(x - y)^2 + 2}{2}\right| = 2(x + y)$
   (d) $\ln\left|\frac{(x + y)^2 + 2}{2}\right| = 2(x - y)$

9. The solution of $y^5 x + y - x\frac{dy}{dx} = 0$ is
   (a) $\frac{y^5}{5} + \frac{y^4}{4x^4} = c$ (b) $\frac{x^4}{4} + \frac{x^5}{5y^5} = c$
   (c) $\frac{y^4}{4} + \frac{y^5}{5x^5} = c$ (d) $\frac{x^5}{5} + \frac{x^4}{4y^4} = c$
   (where c is arbitrary constant)

10. The solution of the equation
$\frac{dy}{dx} + x(x+y) = x^3 (x+y)^3 - 1$ is

(a) $\frac{1}{(x+y)^2} = x^2 + 1 + ce^x$ (b) $\frac{1}{(x+y)} = x^2 + 1 + ce^x$

(c) $\frac{1}{(x+y)^2} = x^2 + 1 + ce^{x^2}$ (d) $\frac{1}{(x+y)} = x^2 + 1 + ce^{x^2}$

(where c is arbitrary constant)

11. Solution of the differential equation
$\left\{\frac{1}{x} - \frac{y^2}{(x-y)^2}\right\} dx + \left\{\frac{x^2}{(x-y)^2} - \frac{1}{y}\right\} dy = 0$ is

(a) $\ln\left|\frac{x}{y}\right| + \frac{xy}{(x-y)} = c$ (b) $\ln|xy| + \frac{xy}{(x-y)} = c$

(c) $\frac{xy}{(x-y)} = ce^{x/y}$ (d) $\frac{xy}{(x-y)} = ce^{xy}$

(where c is arbitrary constant)

12. The real value of n for which the substitution $y = u^n$ will transform the differential equation $2x^4 y \frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation is

(a) 1/2 (b) 1
(c) 3/2 (d) 2

13. The equation of the curve in which the portion of y-axis cut off between the origin and the tangent varies as the cube of the abscissa of the point of contact is

(a) $y = \frac{kx^3}{3} + cx$ (b) $y = -\frac{kx^2}{2} + c$

(c) $y = -\frac{kx^3}{2} + cx$ (d) $y = \frac{kx^3}{3} + \frac{cx^2}{2}$

(k is constant of proportionality)
(where c is arbitrary constant)

14. The equation of the curve for which the square of the ordinate is twice the rectangle contained by the abscissa and the intercept of the normal on x-axis and passing through (2, 1) is

(a) $x^2 + y^2 - x = 0$ (b) $4x^2 + 2y^2 - 9y = 0$

(c) $2x^2 + 4y^2 - 9x = 0$ (d) $4x^2 + 2y^2 - 9x = 0$

15. A curve $y = f(x)$ passes through the point $P(1, 1)$. The normal to the curve at point P is $a(y-1) + (x-1) = 0$. If the slope of the tangent at any point on the curve is proportional to the ordinate at that point, then the equation of the cuve is

(a) $y = e^{ax} - 1$ (b) $y = e^{ax} + 1$

(c) $y = e^{ax} + a$ (d) $y = e^{a(x-1)}$

16. If $y = f(x)$ passing through (1, 2) satisfies the differential equation $y(1+xy)\,dx - x\,dy = 0$, then

(a) $f(x) = \frac{2x}{2-x^2}$ (b) $f(x) = \frac{x+1}{x^2+1}$

(c) $f(x) = \frac{x-1}{4-x^2}$ (d) $f(x) = \frac{4x}{1-2x^2}$

17. The differential equation representing the family of the curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of

(a) order 1, degree 3 (b) order 2, degree 2
(c) order 3, degree 3 (d) order 4, degree 4

18. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$ is

(a) $x\phi(y/x) = k$ (b) $\phi(y/x) = kx$
(c) $y\phi(y/x) = k$ (d) $\phi(y/x) = ky$

(k is arbitrary constant)

19. Let a and b be respectively the degree and order of the differential equation of the family of circles touching the lines $y^2 - x^2 = 0$ and lying in the first and second quadrant, then

(a) $a = 1, b = 2$ (b) $a = 1, b = 1$
(c) $a = 2, b = 1$ (d) $a = 2, b = 2$

20. The integrating factor of the differential equation $\frac{dy}{dx}(x \log_e x) + y = 2\log_e x$ is given by

(a) x (b) e^x
(c) $\log_e x$ (d) $\log_e(\log_e x)$

21. A differential equation associated with the primitive $y = a + be^{5x} + ce^{-7x}$ is

(a) $y_3 + 2y_2 - y_1 = 0$ (b) $y_3 + 2y_2 - 35y_1 = 0$
(c) $4y_3 + 5y_2 - 20y_1 = 0$ (d) none of these

22. A continuously differentiable function $y = f(x), x \in (0, \pi)$ satisfying $y' = 1 + y^2$, $y(0) = 0 = y(\pi)$ is

(a) $\tan x$ (b) $x(x - \pi)$
(c) $(x-\pi)(1-e^x)$ (d) not possible

23. The largest value of c such that there exists a differential function $h(x)$ for $-c < x < c$ that is a solution of $y_1 = 1 + y^2$ with $h(0) = 0$ is

(a) 2π (b) π

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

24. The form of the differential equation of all central conics is

(a) $x = y\frac{dy}{dx}$

(b) $x + y\frac{dy}{dx} = 0$

(c) $x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} = y\frac{dy}{dx}$

(d) none of the above

25. The particular solution of the differential equation $y' + 3xy = x$ which passes through (0, 4) is

(a) $y = 1 - 11e^{-3x^2/2}$ (b) $3y = 1 + 11e^{-3x^2/2}$

(c) $3y = 1 - 11e^{-3x^2/2}$ (d) none of these

Answers

Objective Questions Type I [Only one correct answer]

1. (b)	2. (a)	3. (d)	4. (a)	5. (a)	6. (b)	7. (c)	8. (b)	9. (d)	10. (c)
11. (a)	12. (c)	13. (c)	14. (d)	15. (d)	16. (a)	17. (a)	18. (b)	19. (c)	20. (c)
21. (b)	22. (a)	23. (c)	24. (c)	25. (b)					